

Yices 1.0: An Efficient SMT Solver

AFM'06 Tutorial

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Satisfiability Modulo Theories (SMT)

- ▶ SMT is the problem of determining **satisfiability** of formulas **modulo** background **theories**.
- ▶ Examples of background theories:
 - ▶ linear arithmetic: $x + 1 \leq y$
 - ▶ arrays: $a[i := v_1][j] = v_2$
 - ▶ uninterpreted functions: $f(f(f(x))) = x$
 - ▶ datatypes: $car(cons(v_1, v_3)) = v_2$
 - ▶ bitvectors: $concat(bv_1, bv_2) = bv_3$
- ▶ Example of formula:

$$i - 1 = j + 2, f(i + 3) \neq f(j + 6)$$

Applications of SMT

- ▶ Extended Static Checking
- ▶ Equivalence Checking (Hardware)
- ▶ Bounded Model Checking (e.g., sal-inf-bmc)
- ▶ Predicate Abstraction
- ▶ Symbolic Simulation
- ▶ Test Case Generation (e.g., sal-atg)
- ▶ AI Planning & Scheduling
- ▶ Embedded in Theorem Provers (e.g., PVS)

Yices

- ▶ Yices is an SMT Solver developed at SRI International.
- ▶ Yices is not ICS.
- ▶ It is used in SAL, PVS, and CALO.
- ▶ It is a complete reimplementaion of SRI's previous SMT solvers.
 - ▶ It has a new architecture, and uses new algorithms.
 - ▶ Counterexamples and Unsatisfiable Cores.
 - ▶ Incremental: push, pop, and retract.
 - ▶ Weighted MaxSAT/MaxSMT.
- ▶ Supports all theories in SMT-LIB and much more.

Supported Features

- ▶ Uninterpreted functions
- ▶ Linear real and integer arithmetic
- ▶ Extensional arrays
- ▶ Fixed-size bit-vectors
- ▶ Quantifiers
- ▶ Scalar types
- ▶ Recursive datatypes, tuples, records
- ▶ Lambda expressions
- ▶ Dependent types

Using Yices

- ▶ Starting yices shell: `./yices -i`
- ▶ Batch mode:
 - ▶ Yices format: `./yices ex1.ys`
 - ▶ SMT-LIB format: `./yices -smt ex1.smt`
 - ▶ Dimacs format: `./yices -d ex1.cnf`
- ▶ Increasing verbosity level: `./yices -v 3 ex1.ys`
- ▶ Producing models: `./yices -e ex1.ys`

First Example

```
(define f :: (-> int int))  
(define i :: int)  
(define j :: int)  
(assert (= (- i 1) (+ j 2)))  
(assert (/= (f (+ i 3)) (f (+ j 6))))
```

→ unsat

Check

- ▶ `assert` gets only trivial inconsistencies.
- ▶ `(check)` should be used to test satisfiability.

```
(define x::int)
(define y::int)
(define z::int)
(assert (= (+ (* 3 x) (* 6 y) z) 1))
(assert (= z 2))
(check)
```

→ `unsat`

Extracting Models

▶ `./yices -e ex3.y`

```
(define x::int)
```

```
(define y::int)
```

```
(define f::(-> int int))
```

```
(assert (/= (f (+ x 2)) (f (- y 1))))
```

```
(assert (= x (- y 4)))
```

```
(check)
```

→ sat

```
(= x -2)
```

```
(= y 2)
```

```
(= (f 0) 1)
```

```
(= (f 1) 3)
```

Extracting Unsatisfiable Cores

▶ `./yices -e ex4.ys`

```
(define f::(-> int int))
(define i::int)
(define j::int)
(define k::int)
(assert+ (= (+ i (* 2 k)) 10))
(assert+ (= (- i 1) (+ j 2)))
(assert+ (= (f k) (f i)))
(assert+ (/= (f (+ i 3)) (f (+ j 6))))
(check)
```

→ unsat

```
unsat core ids: 2 4
```

Lemma Learning

- ▶ SMT (and SAT) solvers have a **search engine**:
 - ▶ Case-split
 - ▶ Propagate
 - ▶ Conflict \rightsquigarrow Backtrack
- ▶ Each conflict generates a **Lemma**:
 - ▶ It prevents a conflict from happening again.

Retracting Assertions

- ▶ Assertions asserted with `assert+` can be **retracted**.
- ▶ **Lemmas are reused** in the next call to `(check)`.
 - ▶ Yices knows which lemmas are safe to reuse.

```
(assert+ (= (+ i (* 2 k)) 10))  
(assert+ (= (- i 1) (+ j 2)))  
(assert+ (= (f k) (f i)))  
(assert+ (/= (f (+ i 3)) (f (+ j 6))))  
(check)
```

→ `unsat`

```
(retract 2)  
(check)
```

→ `sat`

Stacking logical contexts

- ▶ (push)
 - ▶ Saves the current logical context on the stack.
- ▶ (pop)
 - ▶ Restores the context from the top of the stack.
 - ▶ Pops it off the stack.
 - ▶ Any changes between the matching `push` and `pop` commands are flushed.
 - ▶ The context is restored to what it was right before the push.
- ▶ Applications (depth-first search):
 - ▶ Symbolic Simulation
 - ▶ Extended Static Checking

Weighted MaxSAT

▶ `./yices -e ex5.y`

```
(assert+ (= (+ i (* 2 k)) 10) 10)
(assert+ (= (- i 1) (+ j 2)) 20)
(assert+ (= (f k) (f i)) 30)
(assert+ (/= (f (+ i 3)) (f (+ j 6))) 15)
(max-sat)
```

→ sat

unsatisfied assertion ids: 4

```
(= i 10) (= k 0) (= j 7) (= (f 0) 11)
(= (f 10) 11) (= (f 13) 12)
```

cost: 10

Type checking

- ▶ By default, Yices assumes the input is correct.
- ▶ It may crash if the input has type errors.
- ▶ You can force Yices to “type check” the input:
 - ▶ `./yices -tc ex1.ys`
 - ▶ Performance penalty.
- ▶ Idea: use `-tc` only when you are developing your front-end for Yices.

Other useful commands

- ▶ `(reset)` – reset the logical context.
- ▶ `(status)` – display the status of the logical context.
- ▶ `(echo [string])` – prints the string `[string]`.

Function (Array) Theory

- ▶ Yices (like PVS) does not make a distinction between arrays and functions.
- ▶ Function theory handles:
 - ▶ Function updates.
 - ▶ Lambda expressions.
 - ▶ Extensionality

Function (Array) Theory (cont.)

▶ Example: `./yices f1.ys`

```
(define A1::(-> int int))
(define A2::(-> int int))
(define v::int) (define w::int)
(define x::int) (define y::int)
(define g::(-> (-> int int) int))
(define f::(-> int int))
(assert (= (update A1 (x) v) A2))
(assert (= (update A1 (y) w) A2))
(assert (/= (f x) (f y)))
(assert (/= (g A1) (g A2)))
(check)
```

→ `unsat`

Lambda expressions

▶ Example: `./yices -e f2.y`

```
(define f::(-> int int))  
(assert (or (= f (lambda (x::int) 0))  
            (= f (lambda (x::int) (+ x 1)))))  
(define x::int)  
(assert (and (>= x 1) (<= x 2)))  
(assert (>= (f x) 3))  
(check)
```

→ sat

```
(= x 2) (= (f 2) 3)
```

Recursive datatypes

- ▶ Similar to PVS and SAL datatypes.
- ▶ Useful for defining: lists, trees, etc.
- ▶ Example: `./yices dt.ys`

```
(define-type list
  (datatype (cons car::int cdr::list) nil))
(define l1::list)
(define l2::list)
(assert (not (nil? l2)))
(assert (not (nil? l1)))
(assert (= (car l1) (car l2)))
(assert (= (cdr l1) (cdr l2)))
(assert (/= l1 l2))
```

→ unsat

Fixed-size bit-vectors

- ▶ It is implemented as a **satellite theory**.
- ▶ Straightforward implementation:
 - ▶ Simplification rules.
 - ▶ **Bit-blasting** for all bit-vector operators but equality.
 - ▶ “Bridge” between bit-vector terms and the boolean variables.

▶ Example: `./yices -e bv.ys`

```
(define b::(bitvector 4))  
(assert (= b (bv-add 0b0010 0b0011)))  
(check)
```

→ unsat

```
(= b 0b0101)
```

Dependent types

- ▶ Useful for stating properties of uninterpreted functions.
- ▶ Alternative to quantifiers.
- ▶ Example: `./yices -e d.ys`

```
(define x::real)
(define y::int)
(define floor::(-> x::real
  (subtype (r::int) (and (>= x r)
    (< x (+ r 1))))))
(assert (and (> x 5) (< x 6)))
(assert (= y (floor x)))
(check)

→ sat
```

```
(= x 11/2) (= y 5) (= (floor 11/2) 5)
```

Quantifiers

- ▶ Main approach: **egraph matching** (Simplify)
 - ▶ Extension for offset equalities and terms.
 - ▶ **Several triggers (multi-patterns)** for each universally quantified expression.
 - ▶ The triggers are fired using a heuristic that gives preference to the most conservative ones.
- ▶ **Fourier Motzkin elimination** to simplify quantified expressions.
- ▶ Instantiation heuristic based on:

What's Decidable About Arrays?,

A. R. Bradley, Z. Manna, and H. B. Sipma, VMCAI'06.

Quantifiers (cont.)

- ▶ Yices may return **unknown** for quantified formulas.
- ▶ The model should be interpreted as a “potential model”.
- ▶ Tuning egraph matching:
 - ▶ `-mi <num>` – Maximum number of quantifier instantiations.
 - ▶ `-mp <num>` – Maximum number of patterns per quantifier.
 - ▶ `-pc <num>` – Pattern generation heuristic (0: liberal, 2: conservative).
- ▶ **Advice: try conservative setting first.**

Quantifiers: example

▶ `./yices q.ys`

```
(define f::(-> int int))
(define g::(-> int int))
(define a::int)
(assert (forall (x::int) (= (f x) x)))
(assert (forall (x::int) (= (g (g x)) x)))
(assert (/= (g (f (g a))) a))
(check)
```

→ `unsat`

C API

- ▶ Yices distribution comes with a C library.
- ▶ Two different APIs:
 - ▶ `yices_c.h`
 - ▶ `yicesl_c.h` (Lite version).

Conclusion

- ▶ Yices is an efficient and flexible SMT solver.
 - ▶ Yices supports all theories in SMT-LIB and much more.
 - ▶ It is being used in **SAL**, **PVS**, and **CALO**.
- ▶ Yices is not ICS.
- ▶ Yices is freely available for end-users.
 - ▶ `http://yices.csl.sri.com`
- ▶ Supported Platforms:
 - ▶ Linux
 - ▶ Windows: Cygwin & MinGW
 - ▶ Mac OSX